NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BAMS	**LEVEL: 7**
COURSE CODE: RAN701S	**COURSE NAME:** REAL ANALYSIS
SESSION: JULY 2022	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

SUPPLEMENTARY /SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR NEGA CHERE
MODERATOR:	PROF FORTUNĖ MASSAMBA

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1

Use the Epsilon- delta ($\epsilon - \delta$) definition of convergence of a sequence to show that $\left(\frac{2n^2}{n^2+1}\right)$ converges to 2. [8]

QUESTION 2

Find $\lim_{n\to\infty}\left(\frac{\cos(n^2+2n+1)}{\sqrt{n}+2}\right)$. [8]

QUESTION 3

3.1. Show directly from the definition that if (x_n) and (y_n) are Cauchy sequences, then

$(x_n - y_n)$ is a Cauchy sequence. [7]

3.2. Prove that a convergent sequence is a Cauchy sequence. [8]

QUESTION 4

Let $x_1 = 2$ and for $n \geq 1$, let $x_{n+1} = 4 - \frac{3}{x_n}$. Assuming that (x_n) converges, find $\lim (x_n)$. [8]

QUESTION 5

5.1. Determine whether the sequence $X = \left(-\frac{2}{1}, \frac{3}{2}, -\frac{4}{3}, \frac{5}{4}, -\frac{6}{5}, \frac{7}{6}, \cdots\right)$ converges or diverges. [8]

5.2. Determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n n^2}{n!}$ converges conditionally or absolutely? [10]

QUESTION 6

Use Epsilon- delta $(\epsilon - \delta)$ definition to show that $\lim_{x\to -2} \frac{2x}{x+4} = -2$. [13]

QUESTION 7

Let $A \subseteq \mathbb{R}$ and let f: $A \to \mathbb{R}$.

7.1. Define what does it mean to say f is uniformly continuous on A? [3]

7.2. Use the definition in (5.1) to show that $f(x) = x^2$ is uniformly on $[-2, 2]$. [10]

QUESTION 8

8.1. Find the fourth Taylor Polynomial centered at 0 for the function $f(x) = \frac{1}{2-x}$. [10]

8.2. Apply the mean value theorem to prove that $|\ln y - \ln x| \leq 4|y - x|$ for $¼ \leq x < y \leq 4$.

[7]

END OF SUPPLEMENTAY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER